Written methods of calculation for Wingrove Primary School

This policy contains the written methods that we ask you to teach at Wingrove Primary School. We aim to encourage consistency and progression throughout our school, therefore the methods chosen are the most suitable for the children in this school. However, you should continuously refer to the framework and be aware of **all** written methods, and if necessary use the appropriate method suitable for a child or group of children.

Our aim is by the end of Key Stage 2, the great majority of children are able to use an efficient written method for each operation with confidence and understanding. Our policy promotes the use of what are commonly known as 'standard' written methods – methods that are efficient and work for any calculations, including those that involve whole numbers or decimals. They are compact and consequently help children to keep track of their recorded steps. Being able to use these written methods gives children an efficient set of tools they can use when they are unable to carry out the calculation in their heads or do not have access to a calculator. We want children to know that they have such a reliable, written method to which they can turn when the need arises.

The challenge for teachers is determining when their children should move on to a refinement in the method and become confident and more efficient at written calculation.

If our expectations are consistent then children's progress will be enhanced rather than limited. Children should be equipped to decide when it is best to use a mental, written or calculator method based on the knowledge that they are in control of this choice as they are able to carry out all three methods with confidence.

Pupils of all ages and abilities should always be encouraged to

look at the calculation/problem and the numbers involved

The size or complexity of the numbers involved often influences the chosen method of calculation. Pupils need to have a firm grounding in 'numbers and the number system'.

decide upon the best method for them to use

The chosen method may be mental calculation with or without jottings, an empty number line, expanded written method, compact written method, calculator or indeed another chosen method that the child is comfortable and accurate in using.

complete the calculation and check the appropriateness of their answer

All pupils should be encouraged to estimate the approximate size of a calculation and use this to check the ‘reasonableness’ of their answer.

The long term aim is for pupils to be able to select an efficient method of their choice that is appropriate for a given task. They will do this by always asking themselves

“Can I do this in my head?”

“Can I do this by using jottings such as an empty number line?”

“Do I need to use a pencil and paper procedure?”

“Do I need a calculator?”

“Have I estimated my answer?”

Pupils need to be given regular opportunities to use and apply calculation methods efficiently to solve a range of problems.

The following pages of this document contain the written calculation stages of development in addition, subtraction, multiplication and division that we have chosen for our school. The stages act as a guide and there is flexibility within the stages for teachers to use their judgement in order to differentiate for the individual pupil abilities within their class.

Written methods for addition of whole numbers

To add successfully, children need to be able to:

- recall all addition pairs to 9 + 9 and complements in 10;
- add mentally a series of one-digit numbers, such as 5 + 8 + 4;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

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Stage 1: The empty number line

- The mental methods that lead to column addition generally involve partitioning, e.g. adding the tens and ones separately, often starting with the tens. Children need to be able to partition numbers in ways other than into tens and ones to help them make multiples of ten by adding in steps.
- The empty number line helps to record the steps on the way to calculating the total.

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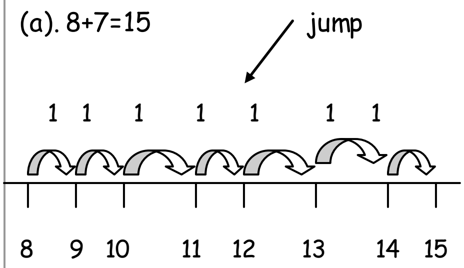
Stage 1

Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10. <u>At Wingrove we will begin by counting in single jumps example (a) before moving onto example (b).</u>

(a). 8+7=15



(b). 8 + 7 = 15

Once the children move onto adding 2- digits or more use stage 2.

Children must be secure with place value to be able to partition accurately.

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Stage 2: Partitioning

- The next stage is to record mental methods using partitioning. Add the tens and then the ones to form partial sums and then add these partial sums.

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Stage 2

Record steps in addition using partitioning. <u>At Wingrove this will be set out as follows:</u>

47+76=

40+70=110

7+ 6= 13

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<tr>
<td>Stage 3: Expanded method in columns
• Move on to a layout showing the addition of the tens to the tens and the ones to the ones separately. Ask children to start by adding the ones digits first always but explain and show by adding tens first you would reach the same answer.
• The addition of the tens in the calculation 47 + 76 is described in the words ‘forty plus seventy equals one hundred and ten’, stressing the link to the related fact ‘four plus seven equals eleven’.
• The expanded method leads children to the more compact method so that they understand its structure and efficiency. The amount of time that should be spent teaching and practising the expanded method will depend on how secure the children are in their recall of number facts and in their understanding of place value.</td>
<td>Stage 3
Write the numbers in columns. At Wingrove we will model example (a) adding the tens first but will also show example (b) adding the units first, emphasising that you get the same answer either way but from now on we will add the units first.

Adding the tens first example (a):
47
+ 76
110
13
123

Adding the ones/units first example (b):
47
+ 76
13
110
123

Use both terms ones and units.</td>
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<tr>
<td>Stage 4: Column method
• In this method, recording is reduced further. Carry digits are recorded above the line, using the words ‘carry ten’ or ‘carry one hundred’, not ‘carry one’.
• Later, extend to adding three two-digit numbers, two three-digit numbers and numbers with different numbers of digits.</td>
<td>Stage 4
4 7   2 5 8   3 6 6
+ 1 7 1 6   + 1 8 1 7   + 4 1 5 1 8
12 3   3 4 5   8 2 4
Column addition remains efficient when used with larger whole numbers and decimals. Once learned, the method is quick and reliable.
Carry digits to be recorded on the doorstep.</td>
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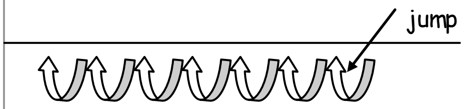
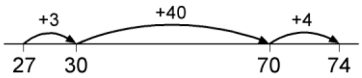
Written methods for subtraction of whole numbers

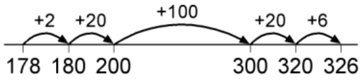
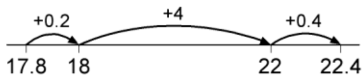
These notes show the stages in building up to using an efficient method for subtraction of two-digit and three-digit whole numbers by the end of Year 4.

To subtract successfully, children need to be able to:

- recall all addition and subtraction facts to 20;
- subtract multiples of 10 (such as 160 – 70) using the related subtraction fact,16 – 7, and their knowledge of place value;
- partition two-digit and three-digit numbers into multiples of one hundred, ten and one in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

Stage 1: Using the empty number line • The empty number line helps to record or explain the steps in mental subtraction. The empty number line is also a useful way of modelling processes such as bridging through a multiple of ten. • The steps can also be recorded by counting up from the smaller to the larger number to find the difference, for example by counting up from 27 to 74 in steps totalling 47. **The notes below give more detail on the counting-up method using an empty number line.**	**Stage 1** Steps in subtraction can be recorded on a number line. The steps often bridge through a multiple of 10. At Wingrove we will be counting back in single jumps (example a) before counting back in larger jumps (example b). (a). 15-7=8  (b).15 – 7 = 8  Count back on the empty number line in stage 1 when focusing on subtraction as ‘take-away’. As soon as children are ready teach ‘find the difference’ also to develop understanding of the counting –up method (stage 2)
The counting-up method • The mental method of counting up from the smaller to the larger number can be recorded using either number lines or vertically in columns. The number of rows (or steps) can be reduced by combining steps. With two-digit numbers, this requires children to be able to work out the answer to a calculation such as 30 + ☐ = 74 mentally.(Only do this with children who can confidently cope)	The 2 examples shown are to be done in stages at Wingrove, initially only the number line will be used this will progress to the number line with the expanded method (shown at the side). 74-27=  74 - 27 3 to make 30 40 to make 70 4 to make 74 47

• With three-digit numbers the number of steps can again be reduced, provided that children are able to work out answers to calculations such as 178 + ☐ = 200 and 200 + ☐ = 326 mentally. • The most compact form of recording remains reasonably efficient.	326-178=  326 - 178 2 to make 180 20 to make 200 100 to make 300 26 to make 326 148 As the children develop confidence with this method they will probably jump in different combinations (e.g. 100+20+6).
• The method can be used with decimals where no more than three columns are required. However, it becomes less efficient when more than three columns are needed. • This counting-up method can be a useful alternative for children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.	22.4-17.8=4.6  22.4 - 17.8 0.2 to make 18 4.0 to make 22 0.4 to make 22.4 4.6
Stage 2: The column method • The compact method should be taught by explaining the value of each digit (partitioning) so that children understand its structure and efficiency. Although the expanded written layout will not be taught it should be explained whilst teaching the compact method.	**Stage 2 –** At Wingrove when teaching the column method staff will be expected to model in the language they use what is happening. Also lots of physical exchanges using equipment will be practised. 6 1 ~~7~~ 4 - 2 7 4 7 6 3 1 ~~7~~ ~~4~~ 1 - 3 6 7 3 7 4 ~~1~~ 6 9 1 ~~7~~ ~~0~~ 1 - 3 6 7 3 3 4

Written methods for multiplication of whole numbers

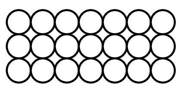
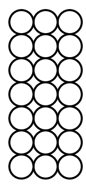
These notes show the stages in building up to using an efficient method for two-digit by one-digit multiplication by the end of Year 4, two-digit by two-digit multiplication by the end of Year 5, and three-digit by two-digit multiplication by the end of Year 6.

To multiply successfully, children need to be able to:

- recall all multiplication facts to 10 × 10;
- partition number into multiples of one hundred, ten and one;
  understand multiplication as repeated addition;
- work out products such as 70 × 5, 70 × 50, 700 × 5 or 700 × 50 using the related fact 7 × 5 and their knowledge of place value;
- add two or more single-digit numbers mentally;
- add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value;
- add combinations of whole numbers using the column method (see above).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

<table>
<tr>
<td>Stage 1: Mental multiplication using partitioning
• Mental methods for multiplying TU × U can be based on the distributive law of multiplication over addition. This allows the tens and ones to be multiplied separately to form partial products. These are then added to find the total product. Either the tens or the ones can be multiplied first but it is more common to start with the tens.</td>
<td>Stage 1
Informal recording
43
40 + 3
↓ ↓ × 6
240 + 18 = 258

Arrays 7x3 or 3x7
 </td>
</tr>
<tr>
<td>Stage 2: The grid method
• As a staging post, an expanded method which uses a grid can be used. This is based on the distributive law and links directly to the mental method. It is an alternative way of recording the same steps.
• It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.</td>
<td>Stage 2
38 × 7 =
<table><tr><td>×</td><td>7</td></tr><tr><td>30</td><td>210</td></tr><tr><td>8</td><td>56</td></tr><tr><td></td><td>266</td></tr></table></td>
</tr>
<tr>
<td>• The grid method may be the main method used by children whose progress is slow, whose mental and written calculation skills are weak and whose projected attainment at the end of Key Stage 2 is towards the lower end of level 4.</td>
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Stage 3: Expanded short multiplication

- The next step is to represent the method of recording in a column format, but showing the working. Draw attention to the links with the grid method above.
- Children should describe what they do by referring to the actual values of the digits in the columns. For example, the first step in 38 × 7 is ‘thirty multiplied by seven’, not ‘three times seven’, although the relationship 3 × 7 should be stressed.
- Most children should be able to use this expanded method for TU × U by the end of Year 4.

Stage 3 - At Wingrove we will multiply units first. When children are secure remove brackets.

```
  38
X  7
  56    ( 8x7)
 210    (30x7)
 266
```

Stage 4: Short multiplication

- The recording is reduced further, with carry digits recorded above the line.
- If, after practice, children cannot use the compact method without making errors, they should return to the expanded format of stage 3.

Stage 4

```
  38
X 57
 266
```

The step here involves adding 210 and 50 mentally with only the 5 in the 50 recorded. This highlights the need for children to be able to add a multiple of 10 to a two-digit or three-digit number mentally before they reach this stage.

Stage 5: Two-digit by two-digit products

- Extend to TU × TU, asking children to estimate first.
- Start with the grid method. The partial products in each row are added, and then the two sums at the end of each row are added to find the total product.
- As in the grid method for TU × U in stage 4, the first column can become an extra top row as a stepping stone to the method below.

Stage 5

56 × 27 is approximately 60 × 30 = 1800.

×	20	7	
50	1000	350	1350
6	120	42	162
			1512
			1

- Reduce the recording further.
- The carry digits in the partial products of 56 × 20 = 120 and 56 × 7 = 392 are usually carried mentally.
- The aim is for most children to use this long multiplication method for TU × TU by the end of Year 5.

At Wingrove when the children are secure remove the brackets.

56 × 27 is approximately 60 × 30 = 1800.

```
   56
 x 27
  392  (56x 7)
 1120  (56x20)
 1512
```

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Stage 6: Three-digit by two-digit products

- Extend to HTU × TU asking children to estimate first. Start with the grid method.
- It is better to place the number with the most digits in the left-hand column of the grid so that it is easier to add the partial products.

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Stage 6

286 × 29 is approximately 300 × 30 = 9000.

<table>
<tr><td>×</td><td>20</td><td>9</td><td></td></tr>
<tr><td>200</td><td>4000</td><td>1800</td><td>5800</td></tr>
<tr><td>80</td><td>1600</td><td>720</td><td>2320</td></tr>
<tr><td>6</td><td>120</td><td>54</td><td>174</td></tr>
<tr><td></td><td></td><td></td><td>8294</td></tr>
<tr><td></td><td></td><td></td><td>1</td></tr>
</table>

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- Reduce the recording, showing the links to the grid method above.
- This expanded method is cumbersome, with six multiplications and a lengthy addition of numbers with different numbers of digits to be carried out. There is plenty of incentive to move on to a more efficient method.

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At Wingrove we will do units first and the carry digit will be recorded above the line.

286
X 29
54 (6x9)
720 (80x9)
1 800 (200x9)
120 (6x20)
1 600 (80x20)
42000 (200x20)
8294

</td>
</tr>
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- Children who are already secure with multiplication for TU × U and TU × TU should have little difficulty in using the same method for HTU × TU.
- Again, the carry digits in the partial products are usually carried mentally.

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286 × 29 is approximately 300 × 30 = 9000.

286
X 29
2 574 (286x9)
51720 (286x20)
8 294

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Written methods for division of whole numbers

<u>These notes show the stages in building up to long division through Years 4 to 6 – first long division TU ÷ U, extending to HTU ÷ U, then HTU ÷ TU, and then short division HTU ÷ U.</u>

To divide successfully in their heads, children need to be able to:

- understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient;
- partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways;
- **recall multiplication and division facts to 10 × 10**, recognise multiples of one-digit numbers and divide multiples of 10 or 100 by a single-digit number using their knowledge of division facts and place value;
- know how to find a remainder working mentally – for example, find the remainder when 48 is divided by 5;
- understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division.

To carry out written methods of division successful, children also need to be able to:

- understand division as repeated subtraction;
- estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92;
- multiply a two-digit number by a single-digit number mentally;
- subtract numbers using the column method.

Stage 1: Expanded division of TU ÷ U and HTU ÷ U

- This method is based on subtracting multiples of the divisor from the number to be divided, the dividend.
- For TU ÷ U there is a link to the mental method.
- As you record the division, ask: 'How many nines in 90?' or 'What is 90 divided by 9?'
- Once they understand and can apply the method, children should be able to move on from TU ÷ U to HTU ÷ U quite quickly as the principles are the same.
- This method, often referred to as 'chunking', is based on subtracting multiples of the divisor, or 'chunks'. Initially children subtract several chunks, but with practice they should look for the biggest multiples of the divisor that they can find to subtract.
- Chunking is useful for reminding

Stage 1 – Example

Wingrove example – chunking first:

```
  2 7
3)8 ²1
```

30 10×3=30

30 10×3=30

21 7× 3=21

27

```
   32 r. 4
6  196
 -  60  10x6
   136
 -  60  10x6
    76
 -  60  10x6
    16
 -  12   2x6
     4   32
```

children of the link between division and repeated subtraction.

- The key to the efficiency of chunking lies in the estimate that is made before the chunking starts.
- Estimating has two purposes when doing a division;
- To help to choose a starting point for the division;
- To check the answer after the calculation.

$$
\begin{array}{rl}
6\overline{)196} & \\
-\underline{180} & 6\times 30 \\
16 & \\
-\underline{\ 12} & 6\times \underline{\ 2} \\
4 & \ \ 32 \\
\text{Answer:} & 32\text{ R }4
\end{array}
$$

Stage 2: Short division TU÷U and HTU÷U

- 'Short' division of TU ÷ U and HTU÷U can be introduced as a more compact recording of the mental method of partitioning.
- Short division of a two-digit or three-digit number can be introduced to children who are confident with multiplication and division facts and with subtracting multiples of 10 mentally, and whose understanding of partitioning and place value is sound.

Stage 2 example for Wingrove:

This is then shortened to:

$$
\begin{array}{r}
2\ 7 \\
3\overline{)8\ {}^{2}1}
\end{array}
$$

The carry digit '2' represents the 2 tens that have been exchanged for 20 units. In the first recording above it is written in front of the 1 to show that 21 is to be divided by 3.

$$
\begin{array}{r}
9\ 7 \\
3\overline{)2\ 9\ {}^{2}1}
\end{array}
$$

The accompanying patter is 'How many threes divide into 80 so that the answer is a multiple of 10?' This gives 20 threes or 60, with 20 remaining. We now ask: 'What is 21 divided by three?' which gives the answer 7.

Stage 3: Long division

The layout on the right, which links to chunking, is in essence the 'long division' method. Recording the build-up to the quotient on the left of the calculation keeps the links with 'chunking' and reduces the errors that tend to occur with the positioning of the first digit of the quotient.

Conventionally the 20, or 2 tens, and the 3 units forming the answer are recorded above the line, as in the second recording.

Stage 3 (At Wingrove)

How many packs of 24 can we make from 560 biscuits? Start by multiplying 24 by multiples of 10 to get an estimate. As 24 × 20 = 480 and 24 × 30 = 720, we know the answer lies between 20 and 30 packs. We start by subtracting 480 from 560.

$$
\begin{array}{rrl}
 & 23\text{ r. }8 & \\
 & 24\overline{)\ 560} & \\
20 & -\underline{480} & 24\times 20 \\
 & 80 & \\
3 & \underline{72} & 24\times 3 \\
 & 8 &
\end{array}
$$

Answer: 23 R 8

In effect, the recording above is the long division method, though conventionally the digits of the answer are recorded above the line as shown below.

$$
\begin{array}{r}
23 \\
24\overline{)\ 560} \\
-\underline{480} \\
80 \\
-\underline{72} \\
8
\end{array}
$$

Answer: 23 R 8

Vocabulary/Glossary

Array	an ordered arrangement of numbers or objects in rows and columns e.g. 1 2 3 or . . . 4 5 6 . . . 7 8 9 . . .
Commutative	addition and multiplication are commutative because when a pair of numbers is added or multiplied the order of the numbers does not matter
Factor	a whole number that divides exactly into another number without leaving a remainder e.g. 4 is a factor of 12 as it divides exactly into 12 12 has 6 factors: 1,2,3,4,6 and 12
Inverse	the opposite operation e.g division is the opposite of multiplication
Multiple	the number made by multiplying together two other numbers. If one number divides exactly into another number, the second is a multiple of the first e.g 28= 4 x 7 so 28 is a multiple of 4
Multiply/ repeated addition	to add a number to itself a given number of times
Partition	to split a number into smaller parts that add to make the number. Numbers are often partitioned into hundreds, tens and ones e.g. 432= 400 + 30 + 2ccccccc
Product	when two or more numbers are multiplied together the answer is the product of those numbers e.g. the product of 3 and 5 is 15 (3x5=15)